A MEASUREMENT OF THE PANOFSKY RATIO WITH A HIGH-ENERGY PAIR SPECTROMETER

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(presented by A. W. Merrison)

The ratio of the reactions

$$\frac{\pi^- + p \rightarrow n + \pi^\circ \rightarrow n + 2\gamma}{\pi^- + p \rightarrow n + \gamma}$$

resulting from negative pions coming to rest in liquid hydrogen has been remeasured by observing the γ-radiation with a large pair spectrometer.

The pair spectrometer is "180° focusing" and has two banks of five scintillation counters each. The phosphors are blocks of plastic scintillator $1'' \times \frac{1}{2}'' \times 3\frac{1}{2}''$; the $3\frac{1}{2}''$ by $1''$ face being presented to the electrons. Adjacent phosphors are separated only by thin aluminium foil which acts as a light seal. The light is transmitted to photomultipliers outside the magnetic field of the spectrometer by perspex light guides.

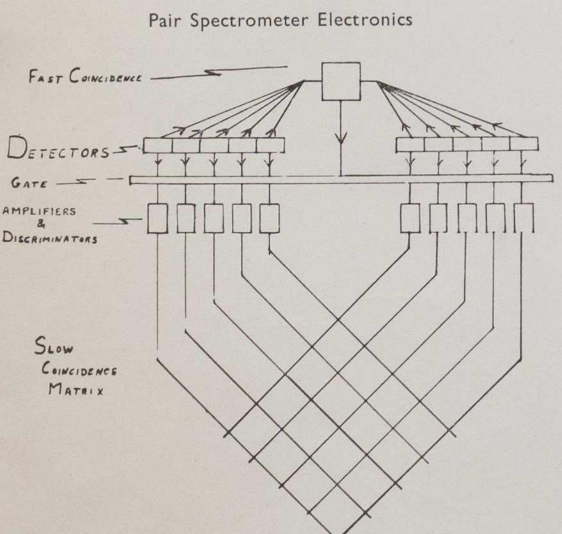

Fig. 1. Simplified block diagram of the electronics.

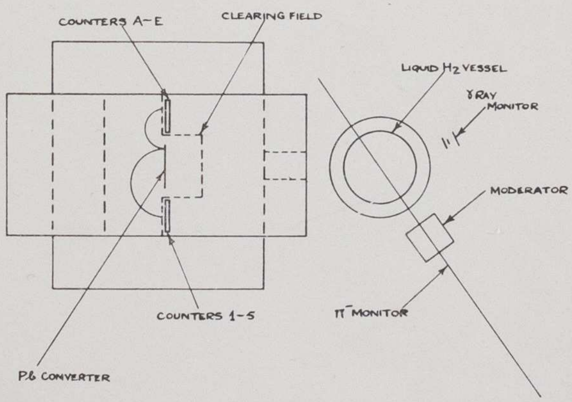

Fig. 2. Experimental arrangement.

A pair is identified by a fast coincidence (20 mμ s) between the two banks of counters, and by pulse height discrimination. The particular pair of counters which has registered the coincidence is then identified and displayed by slow (1 – 10 μ s) electronics.

A simplified block diagram of the electronics is shown in fig. 1. Essentially the two banks of scintillators are linked in a simple fast double coincidence arrangement. If a fast coincidence occurs slow pulses are allowed to go into the counting matrix to show which particular pair of individual scintillation counters are responsible for the coincidence.

The distance between the central phosphors of each bank is 22″, which gives an energy resolution of $4\frac{1}{2}\%$ (full width at half-height).

The magnet has a maximum field of 15,000 gauss across a $3\frac{1}{2}$ ″ gap. The pole pieces are shaped so that there is a clearing field 9″ long before the γ-rays reach the converter. It is electronically stabilised to better than 0.1% and the field for all magnet settings was measured by a nuclear resonance magnetometer to better than 0.1%.

Experimental arrangement (see fig. 2)

The external negative pion beam of the cyclotron was brought to rest in a large spherical vessel full of liquid hydrogen. The beam was monitored by a small scintillation counter before the polythene moderator, and the γ-rays from stopped mesons were monitored by a scintillation counter telescope.

As the whole of the γ-ray spectrum could not be covered at once, four runs were made; one to cover the high-energy peak and three to cover the low energy distribution. For each run the converter thickness was chosen to give the same R.M.S. scattering angle to ensure the same scattering losses into the pole faces. The thicknesses used in this run were 0.003″ to 0.018″, and each converter was 2″ high and 11″ wide.

For all field settings runs were made with the converter removed to give the background. The ratio of real to background counts in the high peak was 7 : 1 and in the low peak about 3 : 1.

The observed counts have to be corrected for 1) the variation with energy of the sensitivity of the spectrometer, 2) the "weighting" of energy channel (some energies see 5 pairs of counters, some only one) and, 3) the variation in pair production cross-section.

A typical γ-ray spectrum obtained is shown in fig. 3. The γ-rays from the two reactions are well-resolved and the "tail" on the resolution curve, due largely to bremsstrahlung creation in the converter, is clearly shown in the top peak, which results of course from a monochromatic γ-ray line. Our present result for the ratio of mesonic to radiative capture is 1.38 ± 0.20 which includes uncertainties in fitting a resolution curve to the results.

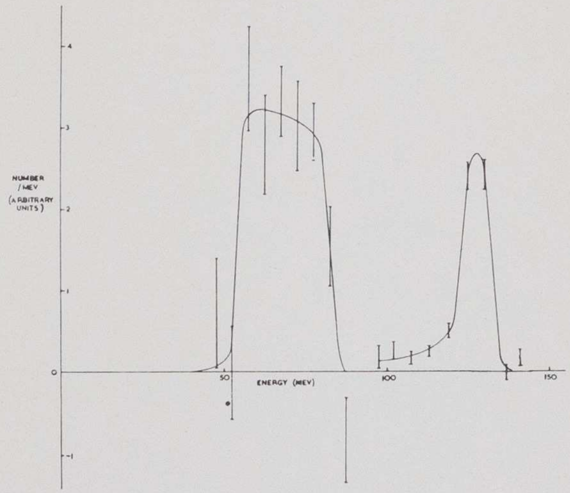

Fig. 3. A typical γ ray spectrum.

The Capture of Negative Pions in Hydrogen and Deuterium (I) Hydrogen

by

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1. Introduction

The experiments carried out by Panofsky, Aamodt and Hadley (1951) on the capture of negative pions in hydrogen and deuterium gave a number of results which were vital to pion physics. They showed that the charged pion has odd parity and gave values for the masses of the neutral and negative pions. Shortly after, Anderson and Fermi (1952) showed, by a simple theoretical argument, that the hydrogen experiment provided a link between meson photo-production experiments and the charge exchange scattering of negative pions by protons.

Panofsky's experiments with hydrogen studied the radiative and mesonic capture of negative pions in mesonic hydrogen atoms, the reactions being

$$\pi^- + p \rightarrow n + \gamma \qquad (1)$$
$$\pi^- + p \rightarrow n + \pi^0 \qquad (2)$$

The gamma-rays from these processes were studied with a high energy pair spectrometer and their energies and relative intensities studied. It has been shown by Wightman (1950) and by Brueckner, Serber and Watson (1951) that by far the most probable fate of a negative pion slowed down in liquid hydrogen is that it should form a mesonic atom and then be captured by the proton from a K-orbit.

Anderson and Fermi pointed out that if one measured the relative rates of reactions (1) and (2) and deduced the absolute rate for radiative capture from meson photo-production data, then one could deduce the absolute rate of the mesonic capture, which corresponds to charge-exchange pion-proton scattering. This gives a value for $(\alpha_3 - \alpha_1)/\eta = 0{\cdot}19 \pm {\cdot}04$ radians (Bethe, Hoffmann, Mesons and Fields 1956) where α_3 and α_1 are the S-wave phase shifts for pion-nucleon scattering in the $T = 3/2$ and $T = \frac{1}{2}$ states. The value deduced by Orear (6) from the best fit to the scattering data is $(\alpha_3 - \alpha_1)/\eta = 0{\cdot}27 \pm {\cdot}03$ radians.

Because of this descrepancy it was clearly important to check the original measurement of Panofsky. We have used a pair-spectrometer with focussing properties rather different to those of Panofsky's instrument to do this. We have also taken advantage of the large external pion beam now available from the Liverpool synchro-cyclotron and have stopped the mesons in this beam in a liquid hydrogen target.

Besides remeasuring the relative rates of reactions (1) and (2) we have also used the Doppler spread in energy of the γ-rays emitted by the neutral pions to make a new and more accurate determination of the $(\pi^- - \pi^0)$ mass difference.

2. Experimental Arrangement.

The general arrangement of apparatus is shown in Fig. 1. Negative pions of mean energy 100 MeV were produced

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Fig. 1 near here

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by bombardment of an internal target in the synchrocyclotron, spiralled out through the fringe field of the cyclotron magnet and were focussed by a strong-focussing octupole magnet and a small wedge magnet. The wedge magnet served also to clear the beam of unwanted particles, and the final intensity of the beam was 8×10^4 pions per sec. with a contamination of 4% μ mesons, and 15% electrons. The beam was monitored by a small scintillation counter and then slowed down in a block of polythene. A proportion of the pions finally stopped in the liquid hydrogen target, the construction of which is shown in Fig. 2. As a check on the

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Fig. 2 near here

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monitoring, a scintillation telescope with a lead converter detected the gamma-rays from mesons stopped in the liquid hydrogen target. Fig. 3 shows the counting-rate in this telescope as a function of polythene absorber thickness.

Fig. 3 near here

The pair spectrometer is "180° focussing" (7), and has two banks of five scintillation counters each. This type of focussing has two main advantages; firstly, the gamma-ray energy is proportional to the detector separation and is independent of the horizontal position of pair creation in the converter; and secondly, angular effects in the resolution are minimised. The phosphors are blocks of plastic scintillator 1" x 3/4" x 3 3/4"; the 3 3/4" x 1" face being presented to the electrons. Adjacent phosphors are separated only by a thin aluminium foil which acts as a light-seal. The light is transmitted to photomultipliers outside the magnetic field of the spectrometer by perspex light guides. The distance between the central phosphors of each bank is 22" which gives an energy resolution of 4½% (see section 5.).

The magnet has a maximum field of 15000 gauss across a 3½" gap. The pole pieces are shaped so that there is a clearing field 9" long before the gamma-rays reach the converter. The field was electronically stabilised to better than 0.1%, and the field for all magnet settings was measured during experimental runs to better than 0.1% by a nuclear resonance magnetometer. The general field shapes were plotted using the magnetoresistance of a bismuth spiral. The deviation of the electron tracks from perfectly circular arcs amounted to 1.4% at the highest field settings. A correction was made for this effect.

A simplified block diagram of the electronics of the pair spectrometer is shown in Fig. 4. Each detector provides a "fast" and a "slow" pulse. The fast pulses from each bank are paralleled to a fast coincidence unit. If a fast coincid-

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Fig. 4 near here

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ence between the two banks of detectors is recorded, a "gate" is opened and the slow pulses are allowed to pass through to amplitude discriminators and an analysing matrix of coincidence circuits which records the pair of counters that have detected the electron pair.

Before all experimental runs, the spectrometer was set up and its performance checked by observing the gammarays from an internal target bombarded by protons.

3. Experimental Procedure.

Because of the limited energy range of the spectrometer, $(1 \pm 1/4)E$ for a mean energy E, each run was broken up into four different magnetic field settings, At each of these field settings the converter thickness was chosen so that the Coulomb scattering of the electrons into the pole pieces remained the same. In this way no corrections need be made for this effect throughout the energy spectrum. The thickest converter used in a normal run was 0.017". A thicker set of converters was used to check resolution and efficiency and this will be discussed below.

The background due to unwanted gamma-rays was measured by removing the hydrogen from the target, and was found to be negligible. The background from other effects was measured by removing the converter. It was found that this background was nearly independent of the magnetic field setting. However as thinner and so less efficient converters were used at the lower energies, the background was most troublesome here.

4. Resolution and Treatment of the Experimental Data.

Although the radiative and mesonic gamma-rays were resolved, it was still essential to know the resolving power of the instrument so that the experimental points could be fitted by the correct resolution curve. The main components of the resolution are (a) finite detector width, (b) energy loss of the electrons in the converter due to bremsstrahlung production, (c) angular effects, which include geometrical effects and horizontal scattering of the electrons, and (d) ionisation losses of the electrons in the converter. These will be discussed briefly.

(a) Finite detector width. The finite widths of a pair of detectors lead to a triangular resolving curve with a full width at half height of 4%.

(b) Bremsstrahlung losses. These were calculated numerically from the Bethe-Heitler theory for electrons (or positrons) carrying off different fractions of the gamma-ray energy, and the results combined assuming all fractions equally probable.

(c) Angular effects. A graphical analysis was first made to determine the gamma-ray illumination of the converter. The resolution was then determined, assuming a Gaussian distribution represented the multiple scattering in the converter.

(d) Ionisation losses. For the thinner converters a small correction to the energy scale was made for this effect. For the thicker ones a Landau distribution was used.

Figure 5 shows the resolution curve of these effects separately and a complete resolution curve from their combination.

The radiative capture peak, which is a natural line spectrum, provides an excellent check on these calculations, (Fig. 6). As a further test the whole experiment

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Figures 5 and 6 near here

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was repeated using a set of converters which was four times as thick as the set used in a normal run.

To make this test more sensitive the two sets of results were subtracted and an analysis carried out on the difference. The results fitted by the calculated resolution curve are shown in Fig. 7.

Fig. 7 near here

Resolution curves for the mesonic capture spectrum were obtained by folding the instrumental resolution curve with the natural spectrum shape, which is rectangular. The experimental results obtained for the mesonic capture peak are shown in Fig. 8.

Fig. 8 near here

Several corrections have to be applied to these curves before one can deduce the relative intensity of the two processes. The major correction is for the variation of the pair production cross-section with energy, for which we used the results of Bethe, Davies and Maximon (1954). Smaller corrections had to be made for gamma-ray absorption in the hydrogen target (3%), dependence of the efficiency on the fractional energy given to the electron (4%), and corrections for electrons or positrons detected in more than one channel (5%). The correction for the overlap of the spectra of the two reactions was $<1\%$.

5. Discussion.

A) The final result for the ratio R of the mesonic capture rate to the radiative capture rate in the hydrogen atom is

$$R\left(\frac{\pi^- + p \rightarrow n + \pi^0}{\pi^- + p \quad n + \gamma}\right) = 1{\cdot}60 \pm 0.17$$

The natural width of the low energy distribution is 28.0 ± 0.7 MeV. This width arises because the gamma-rays from the decay of the neutral pion are emitted while the velocity of the pion is not negligible compared with the velocity of light. This velocity is also of course a measure of the energy released in the mesonic capture and hence of the ($\pi^- - \pi^0$) mass difference. The width observed leads to a value of this mass difference of 9.0 ± 0.3 electron masses. This value is in good agreement with the two previously published values of 10 ± 2 electron masses due to Panofsky (1951) and 8.8 ± 0.6 electron masses due to Chinowsky and Steinberger (1954).

6. Discussion.

Cassels, Fidecaro, Wetherell and Wormald have measured the Panofsky ratio with a total energy Cerenkov counter (private communication). Their result, together with ours and Panofsky's, is given in Table I and

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Table I near here

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the weighted mean calculated. The errors quoted are standard deviations.

One can deduce the absolute rate of the mesonic capture reaction at zero energy and hence $(\alpha_3 - \alpha_1)/\eta$, if one knows the Panofsky ratio and the absolute value of the radiative capture reaction. A value for the latter may be deduced from the photoproduction experiments $\gamma + p \rightarrow \pi^+ + n$ and the ratio at threshold $r_o = \frac{\sigma(\gamma + d \rightarrow \pi^- + p + p)}{\sigma(\gamma + d \rightarrow \pi^+ + n + n)}$.

If one assumes that $r_o = \frac{\sigma(\gamma + n \rightarrow \pi^- + p)}{\sigma(\gamma + p \rightarrow \pi^+ + n)}$ then one can deduce a value for $\sigma(\gamma + n \rightarrow \pi^- + p)$ and hence of its inverse. Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau (1956) give a value for the S-wave part of the reaction $\gamma + p \rightarrow \pi^+ + n$ as $(1.43 \pm 0.02)\,\eta \times 10^{-28}$ cms^2 where η is the pion momentum in units of c. They give also a value for $r_o = 1.87 \pm 0.13$. These results give the S-wave part of the reaction $\pi^- + p \rightarrow n + \gamma$ as $(4.69 \pm 0.33)\,\eta \times 10^{-28}$ cms^2

and hence $(\alpha_3 - \alpha_1)/\eta = 0.25 \pm 0.03$ radians. This agrees very well with the recent Orear value of 0.27 ± 0.015 radians.

The above arguments ignore the charge-dependent corrections which were introduced by Noyes (1956). However it can be shown that if the pion-nucleon interaction has a short range, and there is good evidence for this from the linearity of the S-wave phase shifts and other sources, then such corrections are less than one percent (Dalitz, private communication).

7. Acknowledgements.

A large number of people have contributed to this experiment. The bulk of the pair spectrometer electronics was built by Mr. F.H. Wells and Mr. L. Page of the Atomic Energy Research Establishment, Harwell. Dr. D. Eccleshall loaned us his counting equipment and helped us in many other ways. We have enjoyed the benefit of discussion with Professor J.M. Cassels and members of his group.

Table I

Panofsky etal.	0.94 ± 0.30
Cassels etal.	1.50 ± 0.15
This paper	1.60 ± 0.17
Weighted Mean	1.52 ± 0.11

References:

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9. Chinowsky W. and Steinerberger J. Phys.Rev. 93 586 (1954)
10. Bereventano M. Bernardini G., Carlson-Lee D., Stoppiri G. and Tau L. Il Nuovo Cimento IV No. 2. 323 (1956)

ABSTRACT

A new measurement has been made with a pair spectrometer of the ratio of mesonic and radiative capture of pions stopped in hydrogen. An accurate measurement has also been made of the mass difference of the charged and neutral pions.

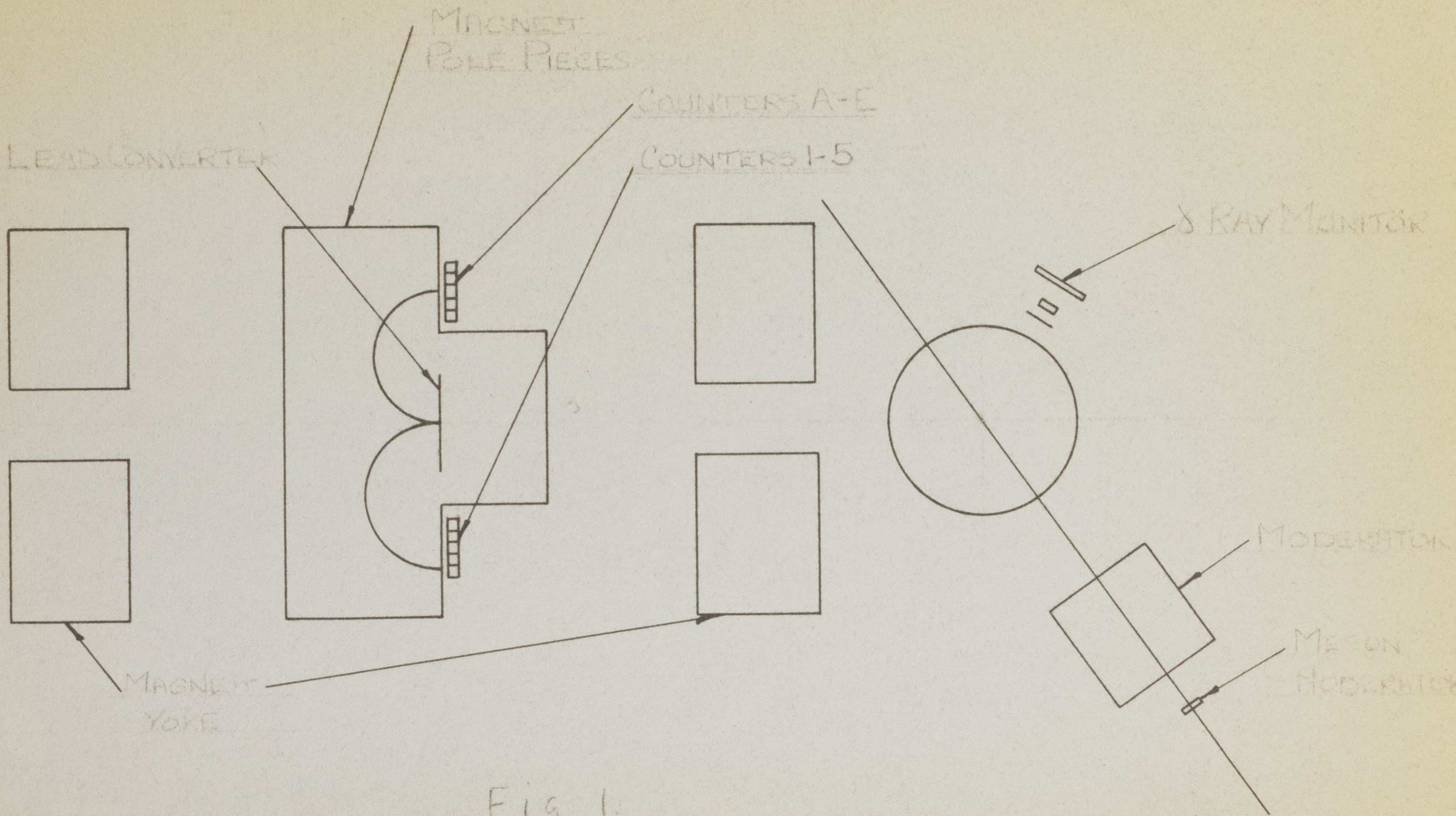

MAGNET
POLE PIECES
COUNTERS A-E
COUNTERS 1-5
LEAD CONVERTER
γ RAY MONITOR
MODERATOR
MESON
MODERATOR
MAGNET
YOKE

Fig 1.

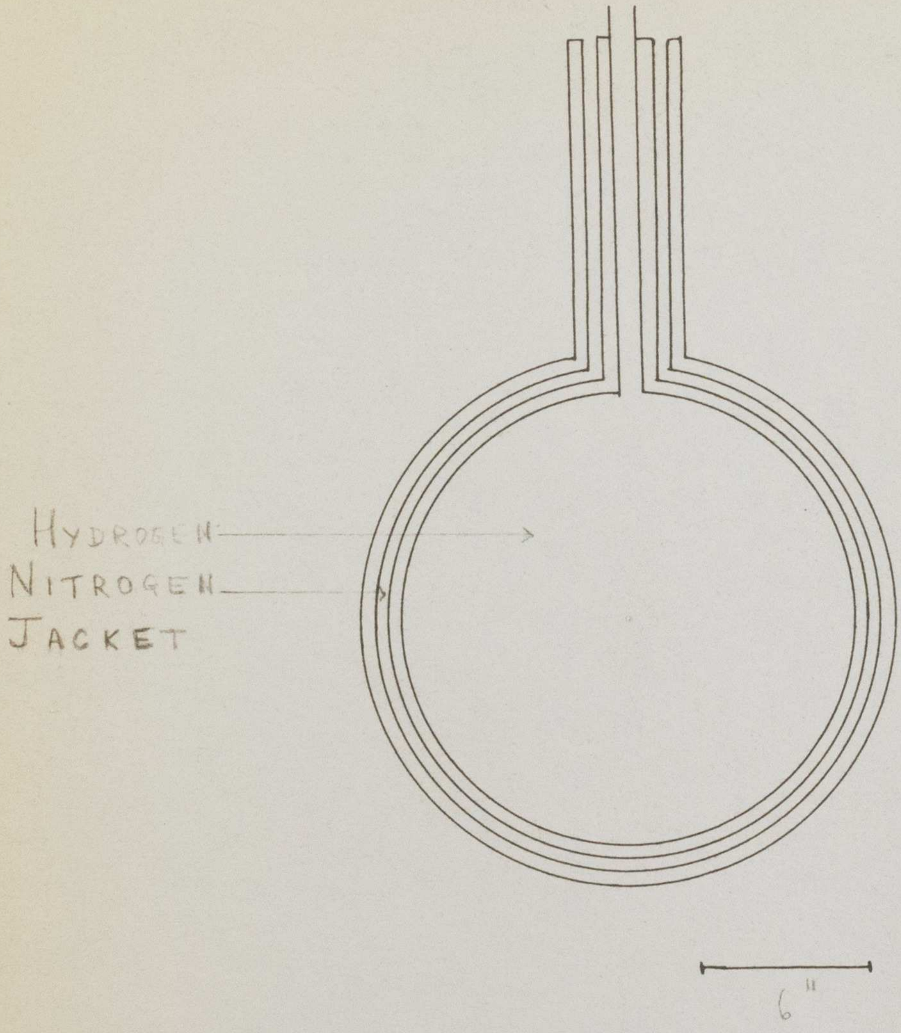
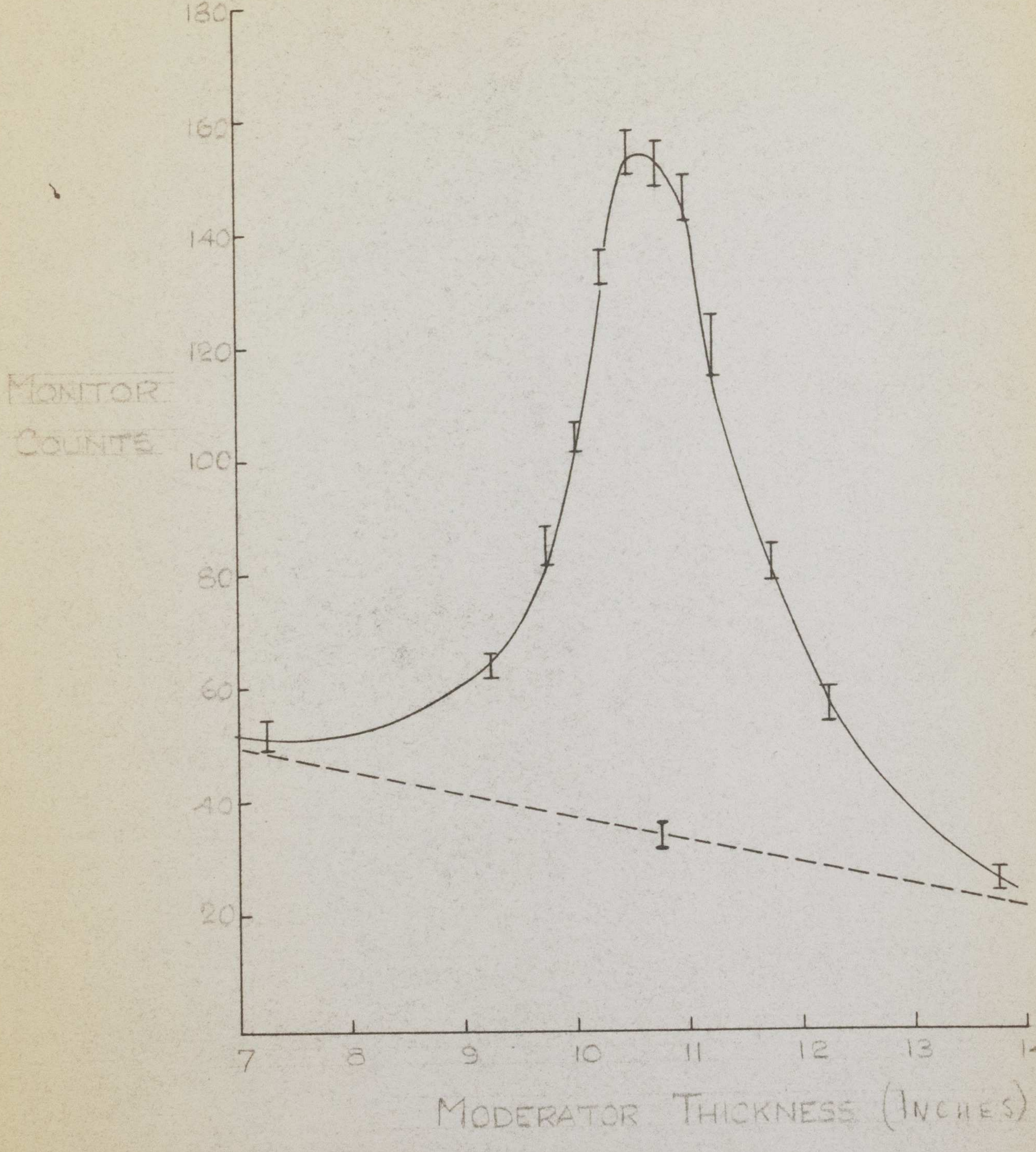


FIG. 2   LIQUID HYDROGEN TARGET



180
160
140
120
100
80
60
40
20
Monitor Counts
7
8
9
10
11
12
13
14
Moderator Thickness (Inches)

Fig 3

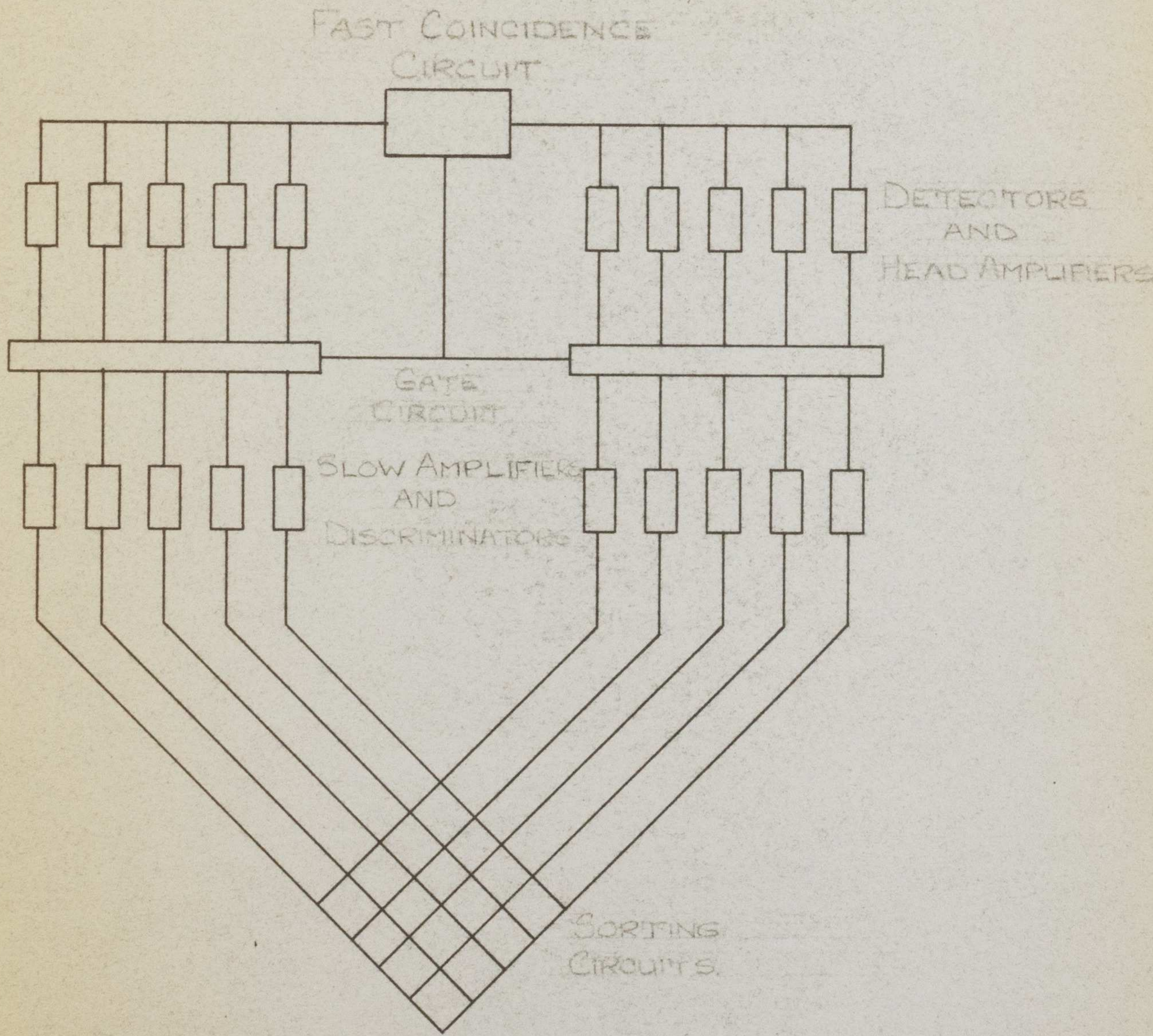
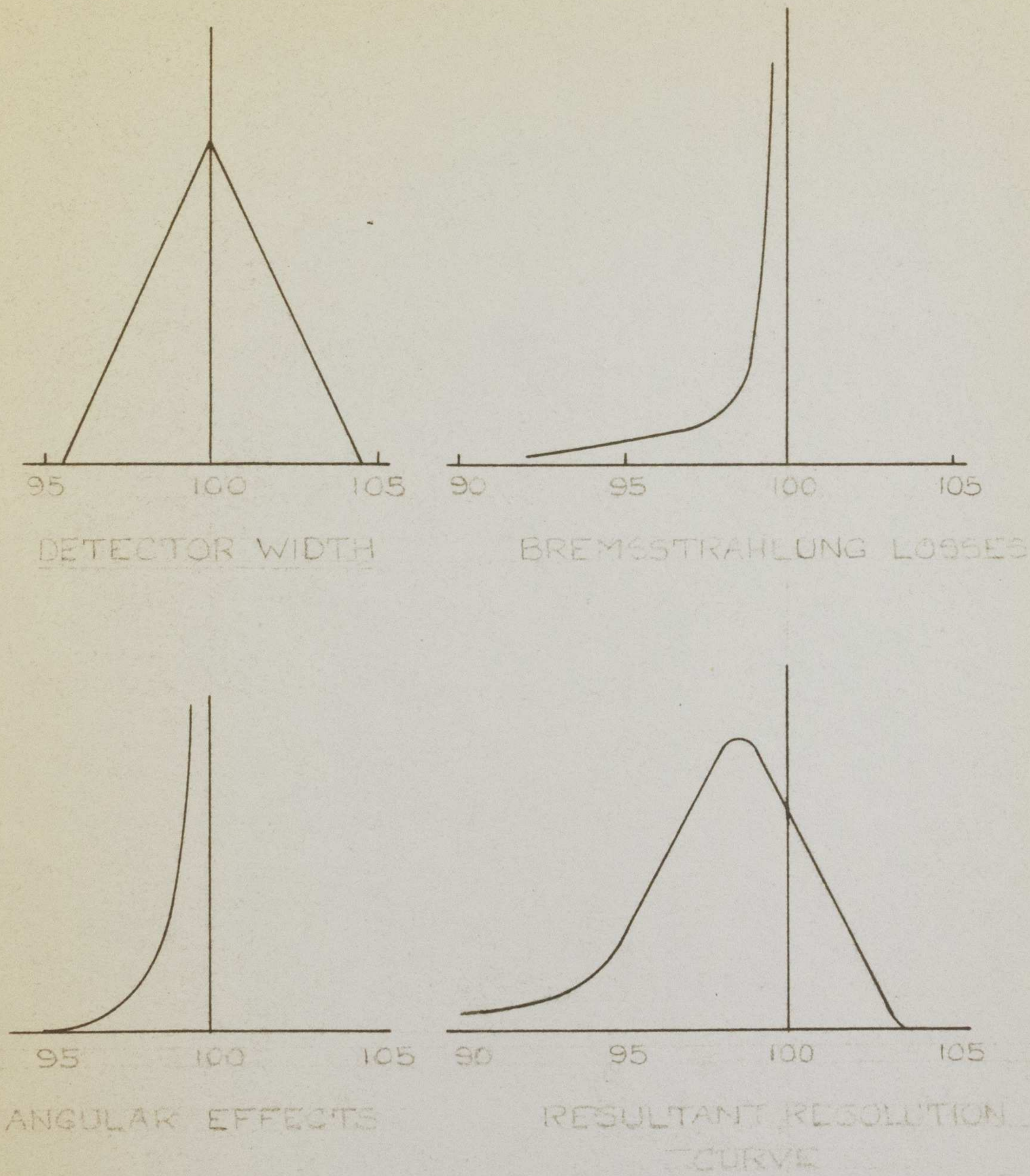

FAST COINCIDENCE CIRCUIT
DETECTORS AND HEAD AMPLIFIERS
GATE CIRCUIT
SLOW AMPLIFIERS AND DISCRIMINATORS
SORTING CIRCUITS.

Fig. 4



MAJOR CAUSES OF RESOLUTION WIDTH
RESULTANT RESOLUTION CURVE

Fig 5

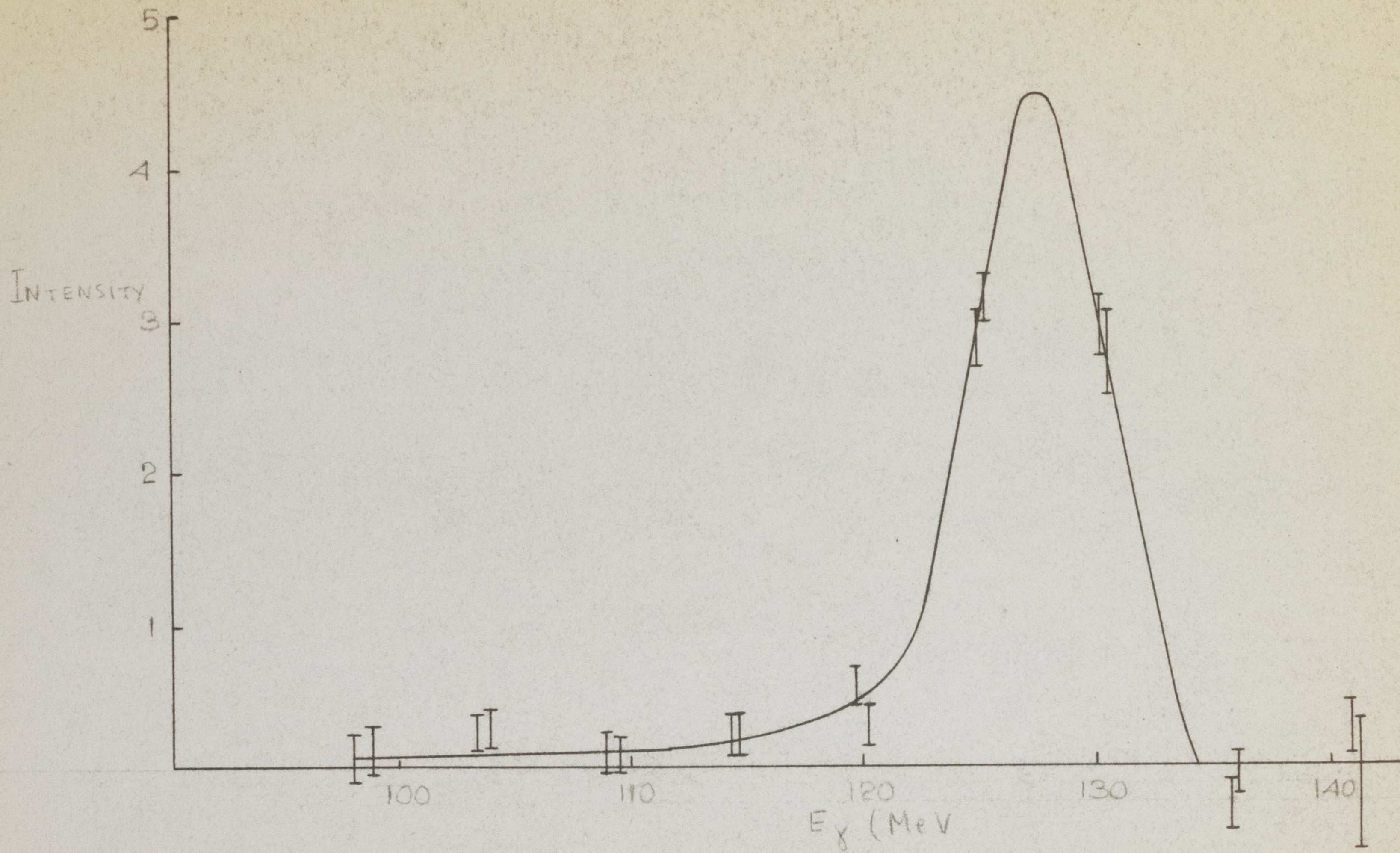


Fig 6

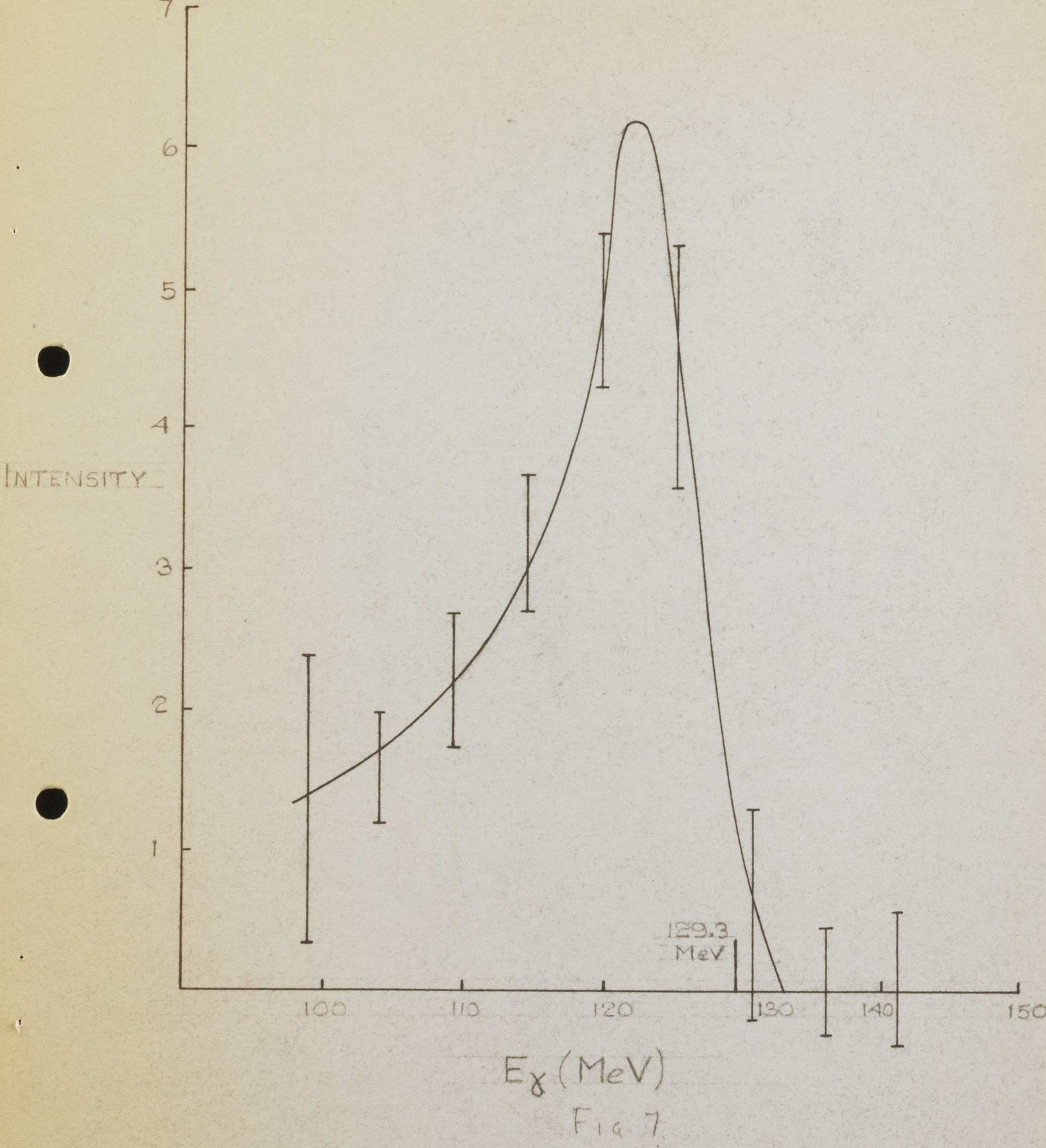
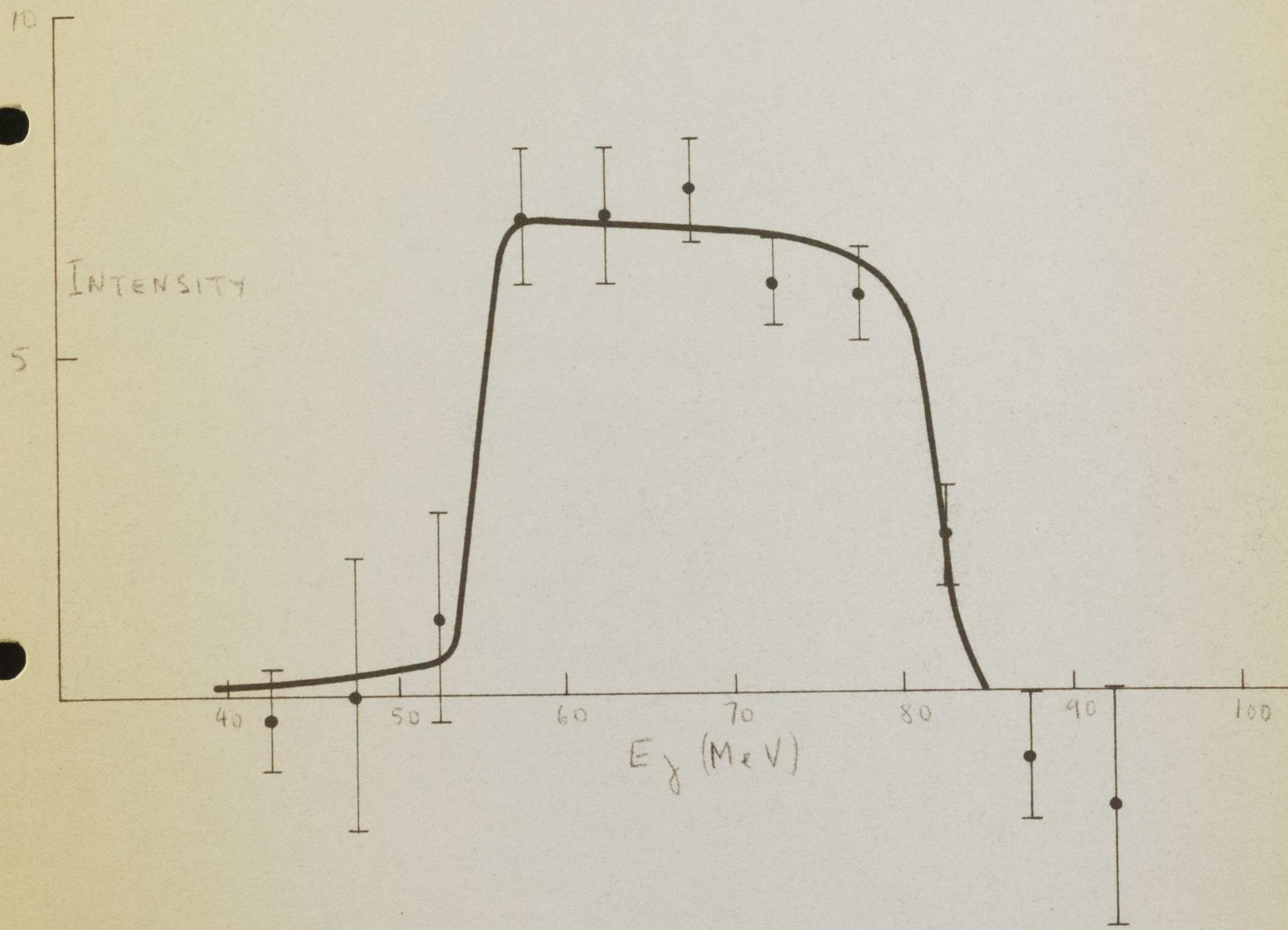


Fig 7



10
INTENSITY
5
40
50
60
70
80
90
100
Eγ (MeV)

The Capture of Negative Pions in Hydrogen and Deuterium (II) Deuterium

by

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1. Introduction

It was shown by the experiments of Panofsky, Aamodt and Hadley[1] that the following reactions could occur when the pion was captured in a mesonic deuterium atom:

$$\pi^- + d \longrightarrow n + n + \gamma \qquad (1)$$

$$\pi^- + d \longrightarrow n + n \qquad (2)$$

The reaction $\pi^- + d \longrightarrow n + n + \pi^0$ was not observed. As gamma-rays only were detected in these experiments, the existence of the non-radiative capture was inferred by observing the relative rate of gamma-ray production in mesonic hydrogen and deuterium atoms. In hydrogen the reactions $\pi^- + p \longrightarrow n + \gamma$ and $\pi^- + p \longrightarrow n + \pi^0$ are the only reactions which can occur and if the relative rates of these is known, then the non-radiative part of the deuterium capture can be deduced. That this reaction occurs at all was perhaps the most important result of Panofsky's experiments, for Ferretti[2] had shown earlier that if the capture takes place in an S-state, as it does in this case,[1][3] then the pion must have odd intrinsic parity. Panofsky obtained for the relative rate of non-radiative capture to radiative capture in deuterium, the value 2.37 ± 0.75.

Brueckner, Serber and Watson[4] have shown too that this ratio can be used to link the photo-production of positive pions to their production in proton-proton collisions. Because the values of the Panofsky ratio of mesonic to radiative capture in the mesonic hydrogen atom which have been determined recently[5][6] are considerably different from Panofsky's original value, it was clearly imperative to repeat the deuterium experiment to see whether there is now any serious discrepancy between the photo-production and proton-proton production data. We have used the external pion beam of the Liverpool synchro-cyclotron and a large pair spectrometer to do this, as we did in our similar experiment with hydrogen.[5] Essentially the experiment compared the number of gamma-rays from pions stopped in identical volumes of liquid hydrogen and liquid deuterium. As in Panofsky's experiment, the ratio of non-radiative to radiative capture in deuterium can then be inferred.

2. Apparatus and Experimental Arrangement

The experimental arrangement differed principally from that used for the hydrogen experiment only in the target. In the present experiment a target vessel was used which could be filled either with liquid hydrogen or liquid deuterium. This target was designed by Dr. S.G.F. Frank of this laboratory. The hydrogen (or deuterium) is condensed from the gas phase into the target in a condenser surrounded by liquid hydrogen. The target also has the very important facility that it may be emptied and then refilled quickly, so that background runs may be made.

The pair-spectrometer was visually positioned so that its collimator channel was in line with the target. The useful height of the target was defined by this collimator while the width was defined by the diameter of the target. It was assumed in the calculations of the energy resolution of the specrometer that this area acted as a uniform source of gamma-rays.

The meson beam was monitored independently by a small scintillation counter and an ionisation chamber in the beam. These monitors agreed to within 2%. To reduce background counts to a minimum a large amount of shielding, in the form of lead bricks and iron-loaded concrete blocks, was placed between the pair-spectrometer detectors and the meson beam.

The experiment was first run with hydrogen and was broken up into runs of one to two hours duration. The total running-time on this part of the experiment was 18 hours, of which about one-third was spent in measuring the background. The change-over to deuterium took three days, and then 25 hours total running-time were spent with the deuterium target, of which $6\frac{1}{2}$ hours were spent measuring the background. The counting rates were approximately 0.7 counts/minute with either hydrogen or deuterium in the target and 0.1 counts/minute with the target empty.

3. Results

The pair-spectrometer was set to cover an energy range from 100 - 140 MeV. The "tails" on both the hydrogen and deuterium spectra extend below this. The tail in the hydrogen case is, of course, due solely to the experimental resolution of the instrument and had been investigated thoroughly in the previous hydrogen experiment. In the deuterium spectrum there is a natural broadening as it represents a three-body decay, and we have used the theory of Watson and Stuart[7] and the experimental results of Phillips and Crowe[8] to estimate this effect. Watson and Stuart's calculations extend down to 120 MeV only. Fortunately, however, most of the gamma-rays are emitted with energies above this. In fitting the results above 120 MeV we used a value of $\hbar c \alpha$ = -12 MeV, where α is the inverse of the scattering length for the singlet neutron-neutron force. The spectrometer was also set in one run to observe the gamma-rays between 40 and 80 MeV but no significant number of gamma-rays was detected here.

The procedure used to determine approximately the shape of the energy spectrum between 80 and 120 MeV was to draw in a set of smooth curves and, after folding the known resolution with this set of curves, to fit the resultant curves to the experimental points. A least-squares fit on each curve then gives the curve which best fits the experimental points, and the standard deviation to be assigned to the total area under the spectrum. This error was $\pm 6.7\%$.

Figure 1 shows the calculated curves for hydrogen and deuterium fitted to the experimental points. The curves

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Fig. 1 near here

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have been normalised to the same number of incoming mesons. The ratio of the areas under these curves multiplied by the ratio of the atomic densities of liquid deuterium and liquid hydrogen gives the ratio of radiative capture in hydrogen to that in deuterium. The result is:

$$\frac{\text{Radiative capture in hydrogen}}{\text{Radiative capture in deuterium}} = \frac{(H)_{\gamma + n}}{(D)_{\gamma + 2n}} = 1.34 \pm 0.13$$

If we now assume that <u>only</u> radiative and non-radiative capture takes place in deuterium, and <u>only</u> radiative and mesonic capture takes place in hydrogen then we can write:

$$\frac{(D)_{2n}}{(D)_{2n+\gamma}} = \frac{(H)_{n+\gamma}}{(D)_{2n+\gamma}} \cdot \frac{(H)_{n+\pi^0}}{(H)_{n+\gamma}} = \frac{(H)_{n+\gamma}}{(D)_{2n+\gamma}} - 1$$

The ratio $\frac{(H)_{n+\pi^0}}{(H)_{n+\gamma}}$ is given in the previous hydrogen paper as 1.50 ± 0.15. Using this value gives the ratio

$$\frac{\text{Non-radiative capture in deuterium}}{\text{Radiative capture in deuterium}} = 2.35 \pm 0.35.$$

This value agrees very well with the 2.37 ± 0.75 found by Panofsky et al.[1]

4. Discussion

The argument connecting the pion photo-production from hydrogen to the production in nucleon-nucleon collisions (see Figure 2) depends on first separating out the S-wave part

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Fig. 2 near here

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of the photo-production. This may be done from the results of Beneventano et al[9] to give a value for $\sigma_s(\gamma + p \rightarrow \pi^+ + n) = (1.43 \pm 0.02)\eta \times 10^{-28}$ cms^2 where η is the pion momentum in units of μc. Beneventano et al also give a value for the ratio of the photo-production of negative pions to positive pions from deuterium at threshold, $r_o = \frac{\gamma + d \rightarrow \pi^- + p + p}{\gamma + d \rightarrow \pi^+ + n + n} = 1.87 \pm 0.13$. If one assumes that $r_o = \frac{\gamma + n \rightarrow \pi^- + p}{\gamma + p \rightarrow \pi^+ + p}$, then one can deduce a value for $\sigma_s(\gamma + n \rightarrow \pi^- + p)$ or, by detailed balancing, its inverse. The above results give for $\sigma_s(\pi^- + p \rightarrow n + \gamma) = (4.69 \pm 0.33)/\eta \times 10^{-28}$ cms^2. To obtain from this a value for the radiative capture cross-section in deuterium, one can use the argument of Brueckner, Serber and Watson which essentially is based on the impulse approximation and calculates the different phase space available for the one neutron in the hydrogen case and the two neutrons in the deuterium case, to arrive at $\frac{\sigma_s(\pi^- + d \rightarrow n + n + \gamma)}{\sigma_s(\pi^- + p \rightarrow n + \gamma)} = \frac{2}{3}$. And so we can deduce a value for $o_s(\pi^- + d \rightarrow n + n + \gamma)$, and hence from the present experiment a value for $\sigma_s(\pi^- + d \rightarrow n + n)$. Again by detailed balancing one arrives at $\sigma_s(n + n \rightarrow \pi^- + d) = (0.17 \pm .03)\eta \times 10^{-27}$ cms^2. This agrees very well for the directly measured value for the charge symmetric reaction[10] of $\sigma_s(p + p \longrightarrow \pi^+ + d) = (0.14 \pm .01)\eta \times 10^{-27}$ cms^2, and shows that any deviations from charge symmetry near threshold in these reactions is small.

5. Acknowledgements

We owe a great deal to Dr. S.G.F. Frank for the loan of his target and the help he gave us during the experimental runs. Dr. R.H. Dalitz gave us very generously a great deal of theoretical advice, particularly on the ratio of radiative capture in deuterium and hydrogen. As in our experiment on hydrogen, we have enjoyed and greatly benefitted from discussion with Professor J.M. Cassels.

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* * *

THE PRODUCTION OF MESONS IN NUCLEON-NUCLEON COLLISIONS

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This introductory talk will be confined to the production of mesons in nucleon-nucleon collisions below 800 Mev. In this region the phenomenological theory for meson production should be useful in interpreting the experimental results, and the multiple production of mesons plays a negligible part. The striking thing about meson production in this energy range is the very rapid rise with energy of the cross-section. Fig. 1 shows the elastic and total cross-sections of protons upon protons. From the production threshold at 290 Mev the cross-section rises until by 800 Mev the nucleon can be looked upon as black with elastic scattering treated as diffraction scattering from an opaque nucleus. This rapid rise can be explained simply if we assume that the matrix elements for meson production are only very slowly varying functions of energy. We can then say that the cross-section for a particular channel is controlled by the statistical weight of the final state; in particular, the statistical weight will be proportional to the volume in momentum space divided by energy. If we have N particles then we can assign the momenta of (N-1) arbitrarily, so the weight is proportional to $\eta^{3(N-1)}/E \sim \eta^{3N-5}$. This applies only to particles emitted in S states; we have an additional η^2 if a particle is emitted in a p-state. In this way we may have excitation functions rising as steeply as η^8.

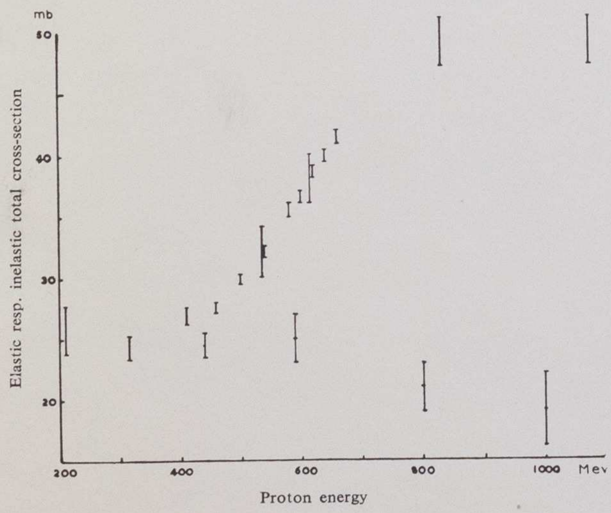

Fig. 1. Elastic (lower points) and inelastic (higher points) total cross sections for proton-proton collisions depending on proton energy in the l.s.

Limiting the field in this way we have 10 possible reactions, not all of which can be studied directly. The neutron upon neutron reactions, with one exception, can be studied only by n-d collisions. The exception is, of course, the inverse reaction $\pi^- + d \rightarrow n + n$. Fortunately if charge symmetry holds in this energy region all the neutron-neutron reactions are duplicated in the proton-proton reactions with interchange of protons with neutrons and positive pions with negative pions.

We have experimental information about all these reactions with exception of $n + n \begin{matrix} \nearrow p + n + \pi^- . \\ \searrow d + \pi^- \end{matrix}$ On these latter reactions some work has been done by Powell and Knapp [1] who have measured angular distributions but no cross-sections unfortunately.

One of the major steps in the simplification of meson-production was taken by Brueckner and Watson [2] in 1951 who showed by assuming that the isotopic spin was conserved in this process, the unbound reactions could be described in terms of 3 production amplitudes. The cross-sections for all reactions can then be most simply expressed in terms of these amplitudes which link given initial and final isospin states of the two nucleons. For example in the reaction $p + p \rightarrow p + p + \pi^\circ$ both initial and final isospin states must be $T = 1$ and so only the cross-section σ_{11} would be involved here. The transition $0 \rightarrow 0$ is forbidden because the meson has $T = 1$. This refers of course only to total cross-sections at a particular incident nucleon energy and a particular emitted meson energy.

We have now our three cross-sections σ_{11}, σ_{10}, σ_{01} to which must be added σ_{10} (d) for the case where the final neutron and proton emerge in the bound state; the deuteron, of course, has $T = 0$. Brueckner and Watson showed that the experimental cross-sections could be formed of linear combinations of these (Table I).*

* See also Fermi [3].

TABLE I

The various possible proton-nucleon reaction expressed in the four fundamental cross-sections σ_{01}, σ_{10}, σ_{11} and σ_{10} (d).

$p + p \rightarrow p + n + \pi^+$	$\sigma_{10} + \sigma_{11}$
$d + \pi^+$	σ_{10} (d)
$p + p + \pi^0$	σ_{11}
$p + n \rightarrow p + n + \pi^0$	$1/2\ (\sigma_{01} + \sigma_{10})$
$d + \pi^0$	$1/2\ \sigma_{10}(d)$
$p + p + \pi^-$	$1/2\ (\sigma_{01} + \sigma_{11})$
$n + n + \pi^+$	$1/2\ (\sigma_{01} + \sigma_{11})$

The factor $1/2$ in all the n-p reactions occurs because this system can be formed from either T = 1 or T = 0 state. Charge independence has been tested by comparing the angular distributions of $p + p \rightarrow \pi^+ + d$ and $p + n \rightarrow \pi^0 + d$ in the experiments of Hildebrand [4] and Wright and Schluter [5]. Within the errors the reactions have identical angular distributions and the n-p reaction has $1/2$ the cross-section of the p-p reaction.

A convenient starting point in the discussion of the production cross-section are the review articles of Rosenfeld [6] and Gell-Mann and Watson [7]. Since then a considerable amount of experimental material has appeared and this will be analysed in terms of the phenomenological models they proposed.

$p + p \rightarrow \pi^+ + d.$

This is by far the best studied reaction because we have the nucleons both initially and finally in known states of isospin ($T = 1 \rightarrow T = 0$) and the final nucleons are in a known angular momentum state $^3S_1$. Also it is particularly easy to study experimentally backwards and forwards.

One has only to guard against observation of

$$p + p \begin{array}{l} \nearrow p + p \\ \searrow n + p + \pi^+ \end{array} \text{ going forwards}$$

$$\text{and } \pi^+ + d \begin{array}{l} \nearrow \pi^+ + d \\ \searrow \pi^+ + n + p \end{array} \text{ going backwards.}$$

If we write down the possible spin states of two nucleons in a T = 1 state with regard to the Pauli principle we have*

$$^1S_0\ ^3P_{0,1,2}\ ^1D_2.$$

If the meson is emitted in an s-state then the parity of the final state is odd and J = 1, and hence the contributing state is $^3P_1$. Similarly if the meson is emitted in a p-state, we have even parity and J = 0, 1 or 2. Hence we can accept $^1S_0$ or $^1D_2$

$$\text{Collecting} \quad ^3P_1 \rightarrow {}^3S_1 s \qquad (1)$$

$$^1D_2 \rightarrow {}^3S_1 p \qquad (2)$$

$$^1S_0 \rightarrow {}^3S_1 p \qquad (3)$$

The s-wave part of the reaction cannot interfere with the p-wave part because they come from singlet and triplet states. So we expect a total cross-section of the form $\sigma = \alpha\eta + \beta\eta^3$.

Before going further we would expect the p-wave part to dominate the reaction because we have mesons emitted in p-states and nucleons left in an s-state. We can also say that we expect the $^1D_2 \rightarrow {}^3S_1p$ to be the dominant p-wave reaction on the following grounds. We know that the ($T = 3/2$, $J = 3/2$) state is very much enhanced in pion-nucleon scattering. Suppose that this is so here. The nucleon which has not emitted a pion must be in an s-state. If we treat this nucleon as an onlooker then we can say the final system is made up of a system with $T = 3/2$, $J = 3/2$ and even parity, and another with $T = 1/2$, $J = 1/2$ and even parity. So for the final system T = 1, 2; J = 1, 2; even parity. And so one would expect the $^1D_2$ state alone among the possible initial states to fulfill these conditions. Whereas (3) has an isotropic angular distribution, one would expect an angular distribution of the form $(1/3 + \cos^2 \theta)$ for $^1D_2 \rightarrow {}^3S_1p$.

It appears in fact that the reaction is swamped by the p-wave part and to measure the s-wave part directly it is necessary to make observations on mesons with less than 20 Mev c.m. energy. This is rather difficult because the cross-sections here are $\sim 1/10$ mb. The only direct experiments are those of Crawford and Stevenson [8]. There is, however, an argument which was proposed by Brueckner, Serber and Watson [9] which is very similar to the argument which leads from the Panofsky ratio for the mesonic and radiative capture of negative pions in hydrogen to the phase shifts in pion nucleon scattering. Panofsky and his colleagues [10], besides observing the gamma-ray spectrum from negative pion capture in hydrogen, also observed that from deuterium. By assuming that the rate of capture from the K orbit is the same, they then deduced that the reaction $\pi^- + d \rightarrow 2n$ was taking place and hence the pion was pseudo-scalar. And so we can start from the experimental result for the ratio

$$R\left(\frac{\pi^- + d \rightarrow 2n}{\pi^- + d \rightarrow 2n + \gamma}\right) = 7/3$$

This measurement has since been repeated by Chinowsky and Steinberger [11].

If we could find the absolute rate for $\pi^- + d \rightarrow 2n + \gamma$ we should know the absolute rate for $\pi^- + d \rightarrow 2n$ and

* Using capital letters to denote the orbital angular momentum state of the two nucleons and small letters that of the meson.

hence by charge symmetry $\pi^+ + d \rightarrow 2p$. Brueckner, Serber and Watson estimate that the ratio

$$\frac{(\pi^- + d \rightarrow 2n + \gamma)}{(\pi^- + p \rightarrow n + \gamma)} = {}^2/_3$$

Then we get at $(\pi^- + p \rightarrow n + \gamma)$ from $\gamma + d \begin{array}{l} \rightarrow n+n+\pi^+ \\ \rightarrow p+p+\pi^- \end{array}$ and $\gamma + p \rightarrow n + \pi^+$. And using the most modern values for these quantities we eventually arrive at $\alpha = 0.2$ mb.

There are experiments [8] which could hope to give a value for α. If one takes their best measured point at $\eta = 0.58$ then one gets within the errors, after taking away the p-wave part, $\alpha = 0.22$.

If one assumes that $\alpha = 0.2$ and then plots $(\sigma - 0.2\,\eta)$ (which should represent the p-wave part) against η^3 then one gets the following curve (fig. 2).

The striking points about this curve are: 1) the flattening off and fall of the cross-section at a proton bombarding energy which would correspond to the pion-nucleon resonance at 170 Mev.; 2) the good linearity of the plot up to $\eta > 1$. The best fit to the total cross-section then is given by $\sigma = 0.2\,\eta + 0.83\,\eta^3$.

The angular distribution information is unfortunately nothing like so accurate (see fig. 3). The Pittsburgh [12] and Moscow [13] points indicate that the value of A remains reasonably constant at approximately 0.23. But there does seem to be genuine disagreement between the Moscow and Pittsburgh points on the one hand and the Liverpool and Chicago points on the other.*

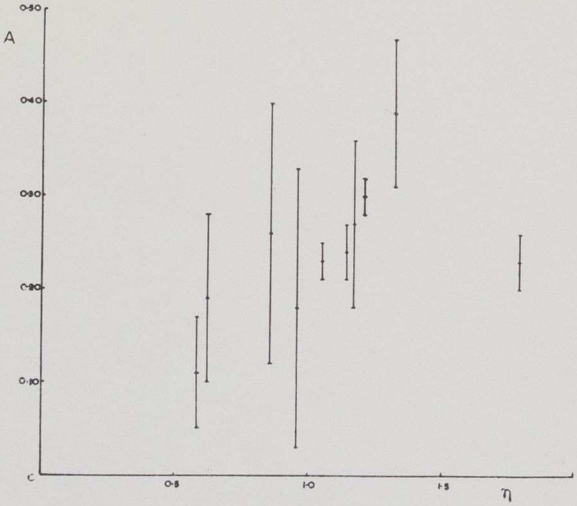

Fig. 3. The coefficient of the isotropic part of the angular distribution in the c.m.s. for the reaction $p+p = \pi^+ + d$ plotted against the maximum available pion momentum.

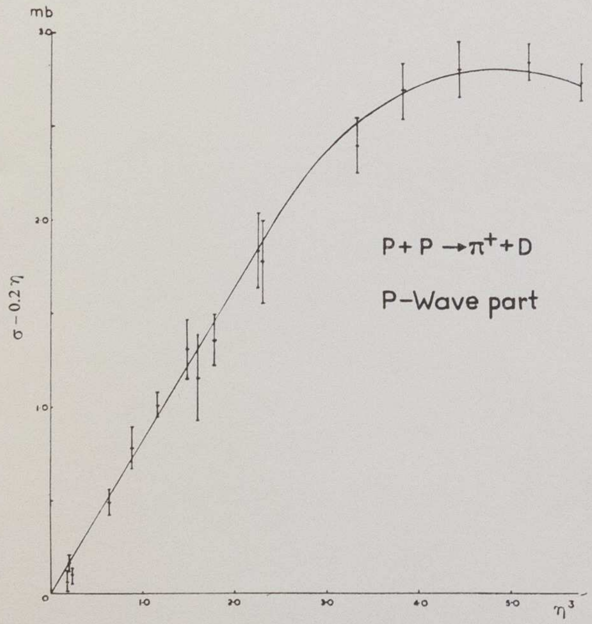

Fig. 2. The p-wave part of the reaction $p+p = \pi^+ + d$ plotted against the third power of the maximum available pion momentum in the c.m.s.

The theory here does not give a very good fit to the experimental points.

$p + p \rightarrow p + p + \pi^0$.

In this reaction we have a final $T = 1$ state so if the nucleons are in an S-state then it must be a ${}^1S_0$ state. So with the meson in a p-state we have a final state $T=1$ $J = 1$ and even parity. The only initial states with even parity are ${}^1S_0$, ${}^1D_2$ which do not have $J = 1$. So we should expect the cross-section of this reaction to be well down on $p\pi^+$ near threshold.

We can write down Ss and Pp and Ps final states with $T = 1$ for the final nucleus.

	Initial	Final	J	Parity	
Ss	${}^3P_0$	${}^1S_0s$	0	−	$\sim \eta^2$
Ps	${}^1S_0$	${}^3P_0s$	0	+	$\sim \eta^6$
	none	${}^3P_1s$	1	+	
	${}^1D_2$	${}^3P_2s$	2	+	

And all the reactions for the class Pp which lead to excitation functions proportional to η^8.

The experimental information up to this year indicated that an excitation function η^8 was probably right. This was based on two measurements, that of Mather and

* The new measurement at Liverpool is on the reaction $\pi^+ + d \rightarrow p + p$ and has been done by D. Eccleshall and A. W. Merrison and yields $\sigma = 1.58 \pm 0.16\,(0.29 \pm 0.02 + \cos^2\theta)$ mb. at 95 Mev pion laboratory energy.

Martinelli [14] and of Marshall et al.[15]. But since then we have the work of Stallwood, Fields, Fox and Kane [16] and Tiapkin and Prokoshkin [17] which has changed the picture completely. This work indicates that all the experimental points may be fitted by a curve of the form $A\eta^6$ in the range 340-660 Mev. This indicates that the process Ps is the dominant one which is a little surprising. Mather and Martinelli pointed out, however, that because a meson was emitted with $l = 1$ with respect to its parent nucleon it does not follow that it has $l = 1$ with respect to the centre of gravity of the nucleons when there is any appreciable relative velocity of the nucleons. The angular distribution, too, according to Tiapkin and Prokoskin is largely isotropic which is what one would expect from a Pp state.

$p + p \rightarrow n + p + \pi^+ \ (\sigma_{10} + \sigma_{11})$

In this reaction the Sp states would be given by

$$^3P_1 \rightarrow {}^3S_1 s$$
$$^1D_2 \rightarrow {}^1S_1 p$$
$$^1S_0 \rightarrow {}^3S_1 p$$

and we can say that the matrix elements connecting given spin states should be the same as with the corresponding deuteron reaction. And the cross-sections now will be given by the deuteron cross-section multiplied by the ratio of numbers of final states in the unbound/bound reactions and by a term $\psi^2(R)/\psi^2{}_d(R)$ where the ψ are the final 2-nucleon wave forms at R, the critical distance for meson production. As this is small ($\approx \hbar/\mu c$) then a zero range approximation is good enough. Not much can be said about the agreement of this theory [18] with experiment at low energies ($\eta < 1$) until the experiments improve. All the earlier work in this field gave experimental values which were about twice as large as the theory predicts but there has been a tendency recently to reduce. In particular the measurement of Stork and Whetstone [19] agrees very well with the theory. The work reported at the Moscow conference seems to suggest that the ratios at higher energies are considerably in excess of the values predicted by this theory.

There are many ways that theory could be in error. In particular at high energies the possibility of large relative velocity of the nucleons is serious. Also it is assumed throughout that the presence of the meson does not upset the nucleon-nucleon force.

The work of Pontekorvo and Selivanov [20] on $n + p \rightarrow \pi^0$ gives good agreement with all the above work.

$n + p \rightarrow \pi^{\pm} \ {}^1/_2(\sigma_{01} + \sigma_{11})$

The only experiments with a bearing on this reaction have been performed by Yodh [21] at Chicago. For nucleons in an S-state we have only the possible reactions

$$^3S_1 \rightarrow {}^1S_0 p$$
$$^3D_1 \rightarrow {}^1S_0 p$$

and for these we expect an excitation function varying as η^4. For all states involving nucleons in a P-state we get a momentum dependence on η^8. If we average Yodh's results for positive and negative pions we get a value for the cross-section of this process at ≈ 400 Mev of 0.16 ± 0.04 mb. If we subtract an extrapolated value for σ_{11} of 0.11 mb for this reaction we arrive at $\sigma_{01} = 0.05$ which is very small indeed.

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The Angular Distribution and Total Cross-Section for the Reaction $\pi^+ + d \longrightarrow p + p$ at 93.5 MeV

by

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1. Introduction

Although a number of experiments on the production of pions in nucleon-nucleon collisions have now been made,[1] the experimental situation is nothing like so definite as in the case of the scattering of pions by nucleons. The simplest reaction to study experimentally is the two-body production process $p + p \rightarrow \pi^+ + d$ or its inverse, but even here the experimental data, although extensive,[2-9] is not sufficiently accurate throughout the whole energy range studied. Because a simple phenomenological theory exists,[10,11] and an attempt has been made to interpret the process with the Chew-Low theory[12] of the pion-nucleon interaction, the present experiments were made. The absorption process $\pi^+ + d \rightarrow p + p$ was studied rather than the direct production process, because with the pions available a rather large centre-of-mass energy could be studied, and hence a search could be made for possible D-wave effects.

2. Experimental Arrangement

The experimental arrangement is shown in Figure 1. The 100 MeV positive pion beam from the Liverpool synchro-cyclotron was monitored by the small scintillation counter C1, which counted about $\frac{1}{10}$ of the beam. The background count in this counter was very small ($<\frac{1}{2}\%$) of the count-rate due to the pions. The beam fell upon the thin-walled target cup which was filled with either heavy or light water. The two protons were detected in the scintillation counters C2, C3, C4. To reduce the background from competing reactions, the absorbers A2 and A3 were placed in front of C3 and C4, and to eliminate the proton contamination in the beam the absorber A1 was placed in front of C1. There are seven times as many of these low energy protons in the beam as pions.

Allowing for the energy lost in the absorber A1, the counter C1 and the target, the average meson interaction energy was 93.5 MeV in the laboratory system, corresponding to 78 MeV in the centre of mass system. This energy is relatively insensitive to the meson distribution across the target. The energy spread in the primary beam and the spread in energy loss in the target result in an energy resolution of $\pm 5\%$. The primary beam energy and energy spread and the contamination of muons and electrons were measured in a separate experiment by a range-curve.

The solid angle of collection was defined by C2, and C3 and C4 were made sufficiently large to allow for multiple scattering in the target scintillators and absorbers. As the energy of the emerging protons varies with angle, the thickness of the absorbers A2 and A3 was chosen so that the energies of the protons entering C3 and C4 was approximately the same at each angular setting. Table I lists the four CM angles used,

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Table I near here

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the angles θ_1 and θ_2 of emission of protons in the laboratory system, the kinetic energies of the protons T_{p1} and T_{p2}, the thickness of the absorbers A2 and A3, and the detection efficiency ε allowing for the absorption of the protons in target, scintillators and absorbers. The detection efficiency was calculated from the absorption cross-sections of Cassels and Lawson.[13]

The photomultiplier outputs were connected to a fast multiple coincidence circuit,[14] and triple coincidences between C2 C3 C4, quadruple coincidences C1 C2 C3 C4, and the singles in C1 were recorded. The angular distributions were measured by the triple coincidence relative to the monitor C1, the absolute cross-section was measured using the quadruple coincidence. In this way very much larger fluxes of mesons are available for the angular distributions, and in this case the whole of the target was illuminated. To do this the distribution of mesons has to be known accurately and for this the beam was scanned in a separate experiment with a very small counter.

3. Results

The results are presented in Tables II and III.

Tables II and III near here

N'_Q represents the counting rate corrected for the following effects

$$N'_Q = \frac{N_Q(1 + x_2)(1 + x_3)}{(1 + x_1)(1 + x_4)\varepsilon R}$$

x_1 = dead-time losses in C1 ($\frac{1}{2}$%)

x_2 = dead-time losses in C2, C3, C4 ($\frac{1}{2}$%)

x_3 = background in C1 and the beam contamination (4%, largely muon contamination)

x_4 = contribution from competing reactions

ε = detection efficiency (Table I)

R = angular resolution correction (up to 16% depending on the angle).

From these results the total cross-section is 8.7 ± 0.6 mb, and the angular distribution is of the form $[(0.293 \pm 0.020) + \cos^2\theta]$. There was no evidence, within the statistics for a $\cos^4\theta$ term.

4. Discussion

Table IV shows the values of the total cross-section for the production reaction ($p + p \longrightarrow \pi^+ + d$) and the constant term A in the angular distribution for various values of η, the meson momentum, in units of μc. The arrow in the first column indicates whether the production reaction ($\longrightarrow$) or its inverse ($\longleftarrow$) was studied.

Table IV near here

That the cross-section and angular distribution fit a crude phenomenological theory can be seen from the following arguments. If we write down the possible spin states of two nucleons in a T = 1 state with regard to the Pauli principle we have*

$$^1S_0 \; ^3P_{0,1,2} \; ^1D_2.$$

If the meson is emitted in an s-state then the parity of the final state is odd and J = 1, and hence the contributing state is $^3P_1$. Similarly if the meson is emitted in a p-state, we have even parity and J = P, 1 or 2. Hence we can accept $^1S_0$ or $^1D_2$

Collecting $^3P_1 \longrightarrow S_1 s$ (1)

$$^1D_2 \longrightarrow {}^3S_1 p \qquad (2)$$

$$^1S_0 \longrightarrow {}^3S_1 p \qquad (3)$$

The s-wave part of the reaction cannot interfere with the p-wave part because they come from singlet and triplet states. So we expect a total cross-section of the form $\sigma = \alpha\eta + \beta\eta^3$.

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* Using capital letters to denote the orbital angular momentum state of the two nucleons and small letters that of the meson.

Before going further we would expect the p-wave part to dominate the reaction because we have mesons emitted in p-states and nucleons left in an s-state. We can also say that we expect the $^1D_2 \longrightarrow {}^3S_1p$ to be the dominant p-wave reaction on the following grounds. We know that the ($T = 3/2$, $J = 3/2$) state is very much enhanced in pion-nucleon scattering. Suppose that this is so here. The nucleon which has not emitted a pion must be in an s-state. If we treat this nucleon as an onlooker then we can say the final system is made up of a system with $T = 3/2$, $J = 3/2$ and even parity, and another with $T = \frac{1}{2}$, $J = \frac{1}{2}$ and even parity. So for the final system $T = 1,2$; $J = 1,2$; even parity. And so one would expect the $^1D_2$ state alone among the possible initial states to fulfil these conditions. Whereas (3) has an isotropic angular distribution, one would expect an angular distribution of the form $(1/3 + \cos^2\theta)$ for $^1D_2 \longrightarrow {}^3S_1p$.

As can be seen from Figure 2 the results on the cross-section are very well fitted by a cross-section of the form $\alpha\eta + \beta\eta^3$ where $\alpha + \beta$ are constants.

Fig. 2 near here

The striking points about this curve are: 1) the flattening off and fall of the cross-section at a proton bombarding energy which would correspond to the pion-nucleon resonance at 170 MeV; 2) the good linearity of the plot up to $\eta > 1$. The best fit to the total cross-section then is given by $\sigma = 0.2\eta + 0.83\eta^2$.

The angular distribution information is unfortunately nothing like so accurate (see Figure 3). The Pittsburgh(15) and Moscow(16) points indicate that the value of A remains

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Fig. 3 near here

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reasonably constant at approximately 0.23. But there does seem to be genuine disagreement between the Moscow and Pittsburgh points on the one hand and the Liverpool and Chicago points on the other.

The theory here does not give a very good fit to the experimental points.

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<u>TABLE I</u>

$\bar{\theta}$	θ_1	θ_2	T_{p1} MeV	T_{p2} MeV	A_2 inches	A_2 inches	ε
90°	78.8°	- 78.8	116.5	116.5	0.526	0	0.884 ± 0.02
70	60	-98.8	131	102	0.426	0.075	0.882 ± 0.02
60	51	-109-3	138	95	0.350	0.128	0.878 ± 0.02
45	37.9	-125.9	146.5	86.5	0.216	0.264	0.873 ± 0.02

TABLE II

C.M. Angle	Heavy Water		Light Water		Net Events from Deuterium per 10^6 Monitor Counts	CM Differential Cross-Section $\times 10^{-28} cm^2$
	Total Triples	Total Monitor Counts	Total Triples	Total Monitor Counts		
90	668	11.34×10^6	205	7.4×10^6	31 ± 3	32.8 ± 3.18
70	653	8.52×10^6	131	4.8×10^6	50 ± 3.8	47.05 ± 3.57
60	1247	10.5×10^6	322	6.87×10^6	71.5 ± 4.2	64.7 ± 3.8
45	3391	18.11×10^6	1001	12.08×10^6	104.2 ± 4.1	90.0 ± 3.6

TABLE III

C.M. Angle	Heavy Water		Light Water		Net Events from Deuterium N_Q per 10^6 Monitor Counts	N'_Q	CM Differential Cross-Section $\times 10^{-28} cm^2$
	Quadruples	Monitor Counts	Quadruples	Monitor Counts			
90	52	11.34×10^6	14	7.4×10^6	2.69 ± 0.83	3.09 ± 0.95	1.93 ± 0.60
70	94	8.52×10^6	12	4.8×10^6	8.52 ± 1.37	9.83 ± 1.59	5.45 ± 0.90
60	142	10.5×10^6	35	6.87×10^6	8.43 ± 1.43	9.84 ± 1.68	5.20 ± 0.90
45	712	31.65×10^6	140	21.59×10^6	16.08 ± 1.00	19.09 ± 1.26	9.46 ± 0.66

TABLE IV

Reaction $p + p \longrightarrow \pi^+ + d$	Lab. beam energy (MeV)	η	σ'_{10}(mb)	A
$\longrightarrow$	310-339	0.377-0.577	-	-
$\longrightarrow$	340	0.582	0.18 ± 0.06	0.11 ± 0.06
$\longleftarrow$	26.5	0.586	0.28 ± 0.05	-
$\longleftarrow$	29	0.621	0.22 ± 0.02	0.19 ± 0.09
$\longrightarrow$	380	0.85	-	0.19 ± 0.06
$\longleftarrow$	48	0.857	0.66 ± 0.07	0.26 ± 0.14
$\longleftarrow$	63	0.957	0.97 ± 0.10	0.18 ± 0.15
$\longrightarrow$	437	1.050	1.23 ± 0.07	0.23 ± 0.02
$\longrightarrow$	460	1.142	1.54 ± 0.16	0.24 ± 0.03
$\longleftarrow$	91	1.171	1.39 ± 0.23	0.27 ± 0.09
$\longleftarrow$	93.5	1.193	1.64 ± 0.11	0.29 ± 0.02
$\longleftarrow$	114	1.321	2.04 ± 0.22	0.39 ± 0.08
$\longrightarrow$	512	1.32	2.1 ± 0.2	-
$\longrightarrow$	560	1.485	2.7 ± 0.15	-
$\longrightarrow$	587	1.565	3 ± 0.15	-
$\longrightarrow$	610	1.635	3.15 ± 0.15	-
$\longrightarrow$	640	1.735	3.2 ± 0.1	-
$\longrightarrow$	657	1.795	3.1 ± 0.1	0.23 ± 0.03
$\longrightarrow$	~ 1000	~ 2.7	< 0.2	-